Gravitational Waves
from
Coalescing Binary Sources

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Abstract

Coalescing binary systems (eg pulsars, neutron stars and black holes) are the most likely sources of gravitational radiation, yet to be detected on or near Earth, where the local gravitational field is negligible and the Poincaré symmetry rules. On the other hand, the general theory of gravitational waves emitted by axially symmetric rotating sources predicts the existence of a non-vanishing news function. The existence of such function implies that, for a distant observer, the asymptotic group of isometries, the BMS group, has a translational symmetry that depends on the orbit periodicity of the source, thus breaking the isotropy o the Poincaré translations. These results suggest the application of the asymptotic BMS-covariant wave equation to obtain a proper theoretical basis for the gravitational waves observations.

1 Gravitational Waves from Binary Sources

Coalescing binary systems composed by pulsars, neutron stars and black holes are considered to be the predominant sources of gravitational waves to be detected by laser interferometers or by resonant mass detectors [1, 2]. Yet, recent reports from various research groups tell that isolated or combined operations of these detectors have failed to produce any evidences of gravitational waves, although they do not exclude a possible success in the near future [3, 4, 5, 6].

On the theoretical side, the most comprehensive study of gravitational waves emitted by axially symmetric rotating systems, which includes binary systems, was presented by Bondi and collaborators in the early 60's. Essentially, the general metric for a rotating axially symmetric gravitational field is expressed in spherical coordinates (u, θ, ϕ) where $u = t - r$ is the retarded time. In these coordinated the source metric takes the general form [7]

$$ds^2 = g_{00}du^2 + 2g_{01}dudr + 2g_{02}dud\theta - g_{22}d\theta^2 - g_{33}d\phi^2 \tag{1}$$

where

$$g_{00} = -A^2r^2e^{2\alpha} + \frac{B}{r}e^{2\beta}, \;\; g_{01} = e^2\beta, \;\; g_{02} = Ar^2e^{2\alpha}, \;\; g_{22} = r^2e^2\alpha, \;\; g_{33} = r^2sin^2\phi\, e^{-2\alpha}$$

and where A, B, α, β are functions of u and θ. Replacing this metric in the vacuum Einstein's equations $R_{\mu\nu} = 0$ and setting proper boundary conditions, the system can be integrated in r and ϕ, with partial integration constants depending on θ and u. From the properties of the Riemann geometry and of the metric (1) these partial integration constants can be reduced to a single function $\mathcal{N}(u, \theta)$ *called the news function*, related to the mass decay of the gravitating system. Taking the expansion of the solution in powers of $1/r$, the Petrov classification shows that *the system emits gravitational radiation if and only if the news function does not vanish*:

$$\mathcal{N}(u, \theta) \neq 0 \tag{2}$$

After the coalescence, when the binary system collapses into a spherically symmetric configuration, this function vanishes [7, 8].

Perhaps of greater importance, was the analysis of the gravitational waves in an asymptotically flat space-time, where the asymptotically flat group of isometries is appropriately defined by the conditions

$$R_{\mu\nu\rho\sigma}\rfloor_{r\rightarrow\infty} = 0, \quad \pounds_\xi g_{\mu\nu}\rfloor_{r\rightarrow\infty} = \xi_{(\mu;\nu)} = 0 \tag{3}$$

These equations define the Bondi-Metzner-Sachs (BMS) group, a semi-direct product of the Lorentz group with the group of supertranslations. The supertranslation operators have the effect of breaking the translational isotropy of the Poincaré group in the sense that *they depend on the rotation angle θ of the source*. This somewhat surprising result produced a distraction to Bondi's original gravitational wave program, suggesting the possibility that elementary particles could be defined in a gravitational environment, by using the classification of the unitary irreducible representations of the BMS group instead of the Poincaré group. However, it was soon found that the BMS is an infinite Lie group whose irreducible unitary representations are impossible to classify in any useful way [9]. Another pursed research was on the nature of the null-cone structure at infinity associated with the BMS symmetry [10].

2 The BMS-Covariant Wave Equation

The standard approach to gravitational waves produced by a distant source assumes that the Minkowski metric is perturbed by the incoming wave as $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$, with $\epsilon^2 << \epsilon$. Replacing this in the vacuum Einstein's equations and using the de Donder gauge, we obtain the well known linear wave equation in Minkowski's space-time

$$\Box^2_\eta \Psi_{\mu\nu} = \eta^{\alpha\beta}\partial_\alpha\partial_\beta\Psi_{\mu\nu} = 0, \qquad \Psi_{\mu\nu} = \eta_{\mu\nu} - \epsilon\frac{1}{2}h_{\mu\nu} \tag{4}$$

On the other hand, since the supertranslations depend on the source rotation, the asymptotically flat BMS-invariant metric also carry informations on that rotation angle.

The general expression of the BMS-invariant metric $\beta_{\mu\nu}$ has been shown to be *not invariant under the Poincaré group*, except in the particular case when the source collapses into a spherically symmetric system [11]. It follows that instead of (4), the correct wave equation will also be invariant under the BMS group, written with the BMS-invariant metric. In order to find this equation we follow a procedure similar to the derivation of the Hartle-Brill-Isaacson high frequency gravitational wave equation [12], where the perturbation of a generic background metric $\gamma_{\mu\nu}$ by an incoming gravitational wave is given by

$$g_{\mu\nu} = \gamma_{\mu\nu} + \epsilon h_{\mu\nu} + \cdots$$

Defining the wave tensor

$$\Psi_{\mu\nu} = h_{\mu\nu} - \epsilon\frac{1}{2}h\gamma_{\mu\nu}, \quad \Psi = \gamma^{\alpha\beta}\psi_{\alpha\beta}$$

Using again the linear condition $\epsilon^2 << \epsilon$ in the vacuum Einstein's equations $R_{\mu\nu}(g) = 0$ for the perturbed metric, we obtain the de Rham gravitational wave equation [13, 14]

$$\Box^2_\gamma \Psi_{\mu\nu} \equiv \gamma^{\alpha\beta}\Psi_{\mu\nu;\alpha\beta} + 2 \overset{(\gamma)}{R}_{\alpha\mu\nu}{}^{\beta}\Psi^{\alpha}_{\beta} + \overset{(\gamma)}{R}_{\mu\alpha}\ \Psi^{\alpha}_{\nu} + \overset{(\gamma)}{R}_{\nu\alpha}\ \Psi^{\alpha}_{\mu} = 0$$

describing the propagation of "high frequency" gravitational waves in an arbitrary background.

In the case of an asymptotically-flat BMS-invariant metric metric $\beta_{\mu\nu}$ we have $\overset{(\beta)}{R}_{\mu\nu\alpha\beta} = 0$, so that the BMS-covariant wave equation becomes simply (notice the covariant derivative with respect to $\beta_{\mu\nu}$)

$$\Box^2_\beta \Psi_{\mu\nu} \equiv \beta^{\alpha\beta}\Psi_{\mu\nu;\alpha\beta} = 0 \tag{5}$$

As it is transparent from this equation, binary systems produce two periodic but distinct effects on the asymptotic detector: The gravitational wave itself described by the wavefunction $\psi_{\mu\nu}$ and the supertranslations included in background metric $\beta_{\alpha\beta}$.

Therefore, even in an asymptotically flat space-time perturbed by an incoming gravitational wave associated with a binary source, the gravitational wave equation to be considered is the asymptotically flat BMS-covariant equation (5). The particular equation (4) holds only in the ideal case where the news function are negligible, in which case no gravitational waves are to be observed. Of course, the angular dependence of the metric $\beta\mu\nu$ requires the knowledge of the most predominant binary system affecting the detectors in the vicinity of the Earth. In principle this can be determined from current astronomical data, as for example from the Sloan Digital Sky Survey (SDSS), as well as from further improvements in the gravitational wave detectors.

The BMS group was obtained in an epoch when the only available gravitational wave detector was Weber's original resonant bar, and when the relevance of binary sources was not yet well understood. Today, astronomical observations and the development of gravitational wave detectors have been dramatically improved, making it possible to verify the effectiveness of the BMS symmetry to the gravitational wave astronomy. In this respect, gravitational wave laser interferometers can be regarded as a sophisticated adaptation of the Michelson-Morley interferometer to measure gravitational waves. The negative result of that experiment eventually led us to a major change in our concepts of symmetry in physics, replacing the Galilean group by the Poincaré group, which is consistent with our present understanding of particle physics and quantum fields. Likewise, if binary sources of gravitational waves are predominant at the location of an asymptotic observer, then the observation of a gravitational wave signature means also an evidence for the BMS group, which is effective at the very long wavelength scale, without compromising the small length scale of particle physics, which remain covariant under the Poincaré group.

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